Probleme geometrie

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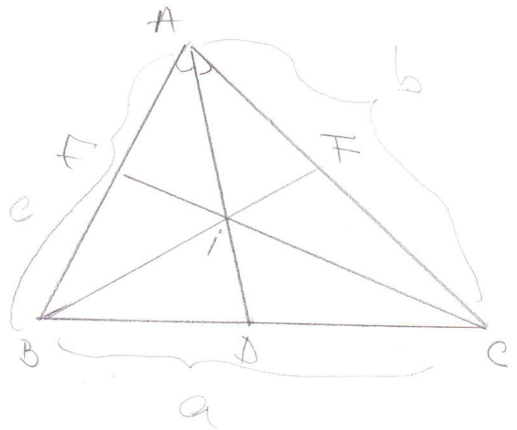
Ip: $\triangle ABC$

$BC = a$

$AC = b$

$AB = c$

I - pct. de int. a bis.



Cl a) $\vec{AI} = \frac{b}{a+b+c}\vec{AB} + \frac{c}{a+b+c}\cdot\vec{AC}$

b) $\vec{PI} = \frac{1}{a+b+c}(a\vec{PA} + b\vec{PB} + c\vec{PC})$

R. Chasles $\Rightarrow$

$\frac{IB}{IF} = k$

$Ai = \frac{1}{k+1}\vec{AB} + \frac{k}{k+1}\vec{AF}$ (1)

$\Rightarrow$ Teorema bisectoarei = fie $\triangle ABC$ și $D \in BC$. $[AD$ - bis $\angle BAC \Rightarrow$

$\Rightarrow \frac{BD}{DC} = \frac{AB}{AC}$

In cazul nostru $\Rightarrow \frac{iB}{iF} = \frac{AB}{AF} = k$

$\Rightarrow \frac{AF}{FC} = \frac{AB}{BC} \Rightarrow \frac{AF}{AC} = \frac{AB}{AB+BC} \Rightarrow \frac{c}{AF} = k$ și $\frac{AF}{b} = \frac{c}{c+a}$

$\Rightarrow k = \frac{c+a}{b}$

$\Rightarrow$ înlocuim în rel. (1) adică $\vec{Ai} = \frac{1}{k+1}\vec{AB} + \frac{k}{k+1}\vec{AF}$

$\vec{Ai} = \frac{1}{\frac{c+a}{b}+1}\vec{AB} + \frac{\frac{c+a}{b}}{\frac{c+a}{b}+1}\cdot\vec{AF}$

$\Rightarrow \frac{b}{c+a+b}\vec{AB} + \frac{c+a}{c+a+b}\vec{AF} = \vec{Ai}$

$\Rightarrow \vec{Ai} = \frac{b}{a+b+c}\vec{AB} + \frac{c}{a+b+c}\vec{AC}$

b) $BD + DC = a$

$\frac{BD}{DC} = \frac{c}{b}$ $\Rightarrow BD = \frac{a\cdot c}{b+c} \Rightarrow \vec{PI} = \frac{1}{k+1}\vec{PD} + \frac{k}{k+1}\vec{PA}$ (2)

$k = \frac{iD}{iA} \Rightarrow \frac{iD}{iA} = \frac{BC}{AC+AB} \Rightarrow \frac{iD}{iA} = \frac{a}{b+c}$

$\vec{PI} = \frac{1}{\frac{a}{b+c}+1}\vec{PD} + \frac{\frac{a}{b+c}}{\frac{a}{b+c}+1}\vec{PA}$ $\Rightarrow \vec{PI} = \frac{1}{1+\frac{c}{b}}\vec{PB} + \frac{\frac{c}{b}}{1+\frac{c}{b}}\vec{PC} \Rightarrow$

$\Rightarrow \vec{PI} = \frac{1}{\frac{1+a}{b+c}}\left[\left(\frac{b}{b+c}\vec{PB} + \frac{c}{b+c}\vec{PC}\right) + \frac{a}{b+c}\vec{PA}\right]$ (A)